1) $f(x) = x^2 - x - 1$ $f(x)$ is decreasing on $(-\infty, \frac{1}{2})$ b/c $f'(x) < 0$

    $f'(x) = 2x - 1$

      $2x - 1 = 0$ $f(x)$ is inc on $(\frac{1}{2}, \infty)$ b/c $f'(x) > 0$

       $x = \frac{1}{2}$

$f'(x)$ $\xleftarrow{\quad - \quad | \quad + \quad}$ $f(x)$ has a local min @ $x = \frac{1}{2}$ b/c $f'(x)$
       $\searrow \frac{1}{2} \nearrow$ Δs signs from $-$ to $+$.
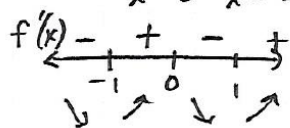
2) $f(x) = 2x^4 - 4x^2 + 1$ $f(x)$ is dec on $(-\infty, -1)$ & $(0, 1)$ b/c $f'(x) < 0$

    $f'(x) = 8x^3 - 8x = 0$ $f(x)$ is inc on $(-1, 0)$ & $(1, \infty)$ b/c $f'(x) > 0$

      $8x(x^2 - 1) = 0$

       $x = 0$ $x = \pm 1$ $f(x)$ has a local min @ $x = -1$ & $x = 1$ b/c

$f'(x)$ $\xleftarrow{- | + | - | +}$ $f'(x)$ Δs signs from $-$ to $+$
     $-1 \quad 0 \quad 1$
     $\searrow \nearrow \searrow \nearrow$

    $f(x)$ has a local max @ $x = 0$ b/c $f'(x)$ Δs
    signs from $+$ to $-$.
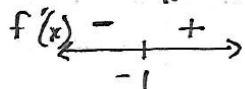
3) $f(x) = x e^x$ $f(x)$ is dec on $(-\infty, -1)$ b/c $f'(x) < 0$

    $f'(x) = x e^x + e^x$ $f(x)$ is inc on $(-1, \infty)$ b/c $f'(x) > 0$

      $= e^x (x + 1) = 0$

       $x = -1$ $f(x)$ has a local min @ $x = -1$ b/c

$f'(x)$ $\xleftarrow{\quad - \quad | \quad + \quad}$ $f'(x)$ Δs signs from $-$ to $+$.
      -1

4) $f(x) = 4x^3 + 21x^2 + 36x - 20$ $f(x)$ is concave down on $(-\infty, -\frac{7}{4})$
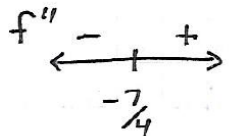
    $f'(x) = 12x^2 + 42x + 36$ b/c $f''(x) < 0$.

    $f''(x) = 24x + 42 = 0$

       $24x = -42$ $f(x)$ is concave up on $(-\frac{7}{4}, \infty)$

        $x = -\frac{7}{4}$ b/c $f''(x) > 0$

f'' $\xleftarrow{\quad - \quad | \quad + \quad}$

   $-\frac{7}{4}$ $f(x)$ has a P.o.I @ $x = -\frac{7}{4}$ b/c $f''(x)$
      Δs signs.

5) $f(x) = 2x^{1/5} + 3$
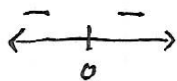
    $f'(x) = \frac{2}{5} x^{-4/5}$ $f(x)$ is concave down on $(-\infty, 0)$ & $(0, \infty)$

           b/c $f''(x) < 0$.
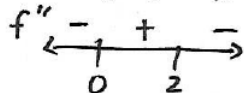
    $f''(x) = -\frac{8}{25} x^{-9/5}$

       $x = 0$

$\xleftarrow{\quad - \quad | \quad - \quad}$
    0

6) $f(x) = -x^4 + 4x^3 - 4x + 1$
$f'(x) = -4x^3 + 12x^2 - 4$
$f''(x) = -12x^2 + 24x = 0$
$\quad\quad -12x(x-2) = 0$
$\quad\quad x = 0 \quad x = 2$

$f''\ \xleftarrow[\ \ \ 0\ \ \ \ \ 2\ \ ]{\ -\ \ \ +\ \ \ -\ }\rightarrow$

$f(x)$ is concave down on $(-\infty, 0)$ & $(2, \infty)$
$\quad$ b/c $f''(x) < 0$

$f(x)$ is concave up on $(0, 2)$ b/c $f''(x) > 0$
$f(x)$ has a P.o.I. @ $x = 0, 2$ b/c
$\quad f''(x)$ Δs signs.

7) $y = xe^x$
$y' = xe^x + e^x$
$y'' = xe^x + e^x + e^x$
$\quad = e^x(x+2) = 0$
$\quad\quad x = -2$

$\xleftarrow[\ \ \ -2\ \ ]{\ -\ \ \ +\ }\rightarrow$
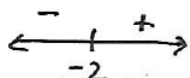
$f(x)$ has a P.o.I @
$x = -2$ b/c f'' Δs signs

8) $f(x) = \arctan x$
$\quad f'(x) = \dfrac{1}{1+x^2} = (1+x^2)^{-1}$

$\quad f''(x) = -1(1+x^2)^{-2} \cdot 2x$

$\quad\quad = \dfrac{-2x}{(1+x^2)^2} = 0$

$\quad\quad\quad x = 0$
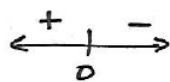
$\xleftarrow[\ \ \ \ 0\ \ \ ]{\ +\ \ \ -\ }\rightarrow$ $\quad f(x)$ has a P.o.I. @
$\quad\quad\quad\quad\quad x = 0$ b/c f'' Δs signs
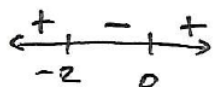
9) $f(x) = x^{4/3} - 4x^{1/3}$

$\quad f'(x) = \dfrac{4}{3}x^{1/3} - \dfrac{4}{3}x^{-2/3}$

$\quad f''(x) = \dfrac{4}{9}x^{-2/3} + \dfrac{8}{9}x^{-5/3} = 0$

$\quad 9x^{5/3}\left(\dfrac{4}{9x^{2/3}} + \dfrac{8}{9x^{5/3}} = 0\right)$
$x = 0$

$\quad\quad 4x + 8 = 0$
$\quad\quad\quad x = -2$

$\xleftarrow[\ \ -2\ \ \ \ \ 0\ \ ]{\ +\ \ \ -\ \ \ +\ }\rightarrow$

$f(x)$ has P.o.I @ $x = -2, 0$
b/c f'' Δs signs

a) $f(x)$ has a local max @ $x = -3$
& $x = 4$ b/c $f'(x)$ Δs signs from $+$ to $-$

b) $f(x)$ has a P.o.I @ $x = -4, -1, 2$
b/c $f''(x)$ Δs signs.

c) $f(x)$ is concave up on $(-5, -4)$, $(-1, 1)$
$(1, 2)$ b/c $f'' > 0$

d) f has a positive slope on
$(-5, -3)$ & $(1, 4)$ b/c $f'(x) > 0$.